International Journal of Computing Science and Information Technology, **2014, Vol.2(2), 15-19**
ISSN: 2278-9669, April 2014 (http://ijcsit.org)

Optimization of trapezoidal balanced Transportation problem using zero-suffix and Robust Ranking Methodology with fuzzy demand and fuzzy supply models

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Abstract

The Analysis of 'Fuzzy counter parts' of linear programming problems of some special structure, for example problems of flows in networks Transportation problems and so on, appears to be an interesting task. The following model considered a transportation problem with fuzzy supply values of the supplier and with fuzzy demand values of the receivers. The objectives of this paper to find the minimum transportation cost of some commodities through capacitate network, when the supply and demand of nodes and the capacity and cost edges are representing trapezoidal fuzzy numbers. We proposed a Robust Ranking techniques and zero-suffix method for solving fuzzy balanced transportation problem with fuzzy demand and fuzzy supply.

Keywords: Robust Ranking method, Zero-Suffix method, Fuzzy demand, Fuzzy supply, Trapezoidal.

1. Introduction

The standard transportation model seeks to find a transportation plan for a single commodity from a number of sources to a number of destinations. The data in the model includes:

(1) The amount of supply at each source and the demand at each destination.

(2) The unit transportation cost of commodity from each source to each destination.

A destination may receive its demand from many sources. The objective is to determine the shipping plan, to meet all demands but not exceed any supply, to reduce the total transportation cost.

Assume there are m sources and n destinations. Let xij be the amount to be shipped from source i to destination j , $1 \le i \le m$, $1 \le j \le n$, xij will be an integer greater than, or equal to zero.

Now the linear programming model representing the transportation problem is given below

$$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints,

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad j = 1,2,\ldots\ldots\ldots n$$

$$\sum_{j=1}^{n} x_{ij} \le a_i \quad i = 1,2,\ldots\ldots\ldots m$$

And xij ≥ 0 and xij is integer . For the model to be feasible, this must have the total supply is equal to the total demand.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

1.1 Fuzzy transportation problem

All of the parameters Cij , ai, bj to be fuzzy showing any uncertainty in their values. However, the xij will be crisp because if we allowed them to be fuzzy we would have to defuzzify them at the end to obtain a feasible shipping plan.

Let $\widetilde{c_{ij}}$ a trapezoidal fuzzy number representing the cost of sending one unit from source i to destination j. $\widetilde{A_i}$ is a trapezoidal fuzzy number for the amount at source i and $\widetilde{B_j}$ is another trapezoidal fuzzy number for the demand at destination j. The fuzzy optimization problem to solve is

$$\text{Min } z = \sum_{i=1}^{n} \sum_{i=1}^{m} \widetilde{C_{ij}}\, x_{ij}$$

Subject to the constraints,

$$\sum_{i=1}^{m} x_{ij} \geq \widetilde{B_j} \qquad j = 1, 2, \ldots n$$

$$\sum_{j=1}^{n} x_{ij} \leq \widetilde{A_i} \qquad i = 1,2, \ldots m$$

And xij ≥ 0 and xij integer . For the model to be feasible we must have the total supply is equal to the total demand.

$$\sum_{i=1}^{m} \widetilde{A_i} = \sum_{j=1}^{n} \widetilde{B_j}$$

The transportation problem is one of the earliest applications of linear programming problems. The appearance of randomness and imprecision in the real work is unavoidable due to some unexpected situations. There are some cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors are studied by some of the researchers in [1,2].

A fuzzy transportation problem `is a problem in which the transportation cost, supply, and demand quantities are fuzzy quantities. The aim of the fuzzy transportation is to be determined shipping schedule that minimizes the total fuzzy transportation cost while satisfying the fuzzy supply and demand limits. In real world applications, all the parameters are of the transportations problems may not be known precisely due to uncontrollable factors. Fuzzy numbers are introduced by Zadeh may represents this data[3] . Zimmermann showed that solution obtained by fuzzy linear programming are always efficient [4] .

Chanas and Kuchta projected the notion of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution[5]. Saad and Abbas discussed the solution algorithm for solving the transportation problem in fuzzy environment [7].Dinagor and Palanivel investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers [6]. R.Nagarajan and A.Solairaju [8] presented an algorithm for solving fuzzy assignment problems using Robust ranking technique with fixed fuzzy numbers.

In this paper we applied a Robust Ranking techniques and zero-suffix method for solving fuzzy balanced transportation problem with fuzzy demand and fuzzy supply when the supply, demand, and cost edges are representing trapezoidal fuzzy numbers. Finally, feasibility of the proposed study is checked with a numerical example.

2. Preliminaries

Zadeh, first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life in 1965[3].

2.1 Definition

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (i,e) A = {(x, μA(x) ; x Є X}, Here μA: X → [0,1] is a mapping called the degree of membership function of the fuzzy set A and μA(x) is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition: (Trapezoidal fuzzy number): For a trapezoidal number A(x), it can be represented by A (a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ (d-x)/(d-c), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

2.3 Definition: (α-cut of a trapezoidal fuzzy number): The α-cut of a fuzzy number A(x) is defined as A (α) = {x : μ(x) ≥ α, α Є [0,1]} Addition of two fuzzy numbers can be performed as

$$(a1,b1,c1) + (a2,b2,c2) = (a1+a2,\ b1+b2,\ c1+c2)$$

Addition of two trapezoidal fuzzy numbers can be performed.

3. Robust's Ranking Techniques

Robust's ranking technique [8] which satisfies costs, linearity, and additives properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust's Ranking Index is defined by

R (ã) = $\int_0^1 0.5\,(a_\alpha^L, a_\alpha^U)d\alpha$, where (a_α^L, a_α^U) is the α – level cut of the fuzzy number ã.

In this paper we use this method for ranking the objective values. The Robust's ranking index R (ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property:

4. Zero Suffix Method

The zero suffix method proceeds as follows.

Step 1 : Construct the transportation table for the given TP and check the balanced condition. If not, convert it into balanced one.

Step 2 : Subtract each row entries of the transportation table from the row minimum.

Step 3 : subtract each column entries of the resulting transportation table after using the Step 2 from the column minimum.

Step 4 : In the reduced cost matrix there will be atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

Therefore S= {Add the costs of nearest adjacent sides of zero No. of costs added}

Step 5 : Choose the maximum of S, if it has one maximum value then supply to that demand corresponding to the cell. If it has more equal values then select {ai,bj } and supply to that demand maximum possible.

Step 6 : After the above step, the exhausted demands (column) or supplies (row) are to be trimmed. If ai = bj , cross out either ith row or jth column but not both. The resultant matrix must possess atleast one zero is each row and each column, else repeat step2 and step3.

Step 7 : Repeat Step 4 to Step 6 until the optimal cost is obtained.

5. Numerical Example: (Trapezoidal Fuzzy Number)

A company has four sources S1, S2, S3, S4 and destinations D1, D2, D3, D4. The fuzzy transportation cost for unit quantity of product from ith sources jth destinations is Cij

Where Cij,

$$\begin{pmatrix} (4, 4.5, 5.5, 6) & (6, 6.5, 7.5, 8) & (7, 7.5, 8.5, 9) \\ (3,3.5, 4.5, 5) & (3,3.5, 4.5, 5) & (5, 5.5, 6.5, 7) \\ (5, 5.5, 6.5, 7) & (6, 6.5, 7.5, 8) & (6, 6.5, 7.5, 8) \end{pmatrix}$$

Fuzzy availability of the product at source are (64 64.5 65.5 66) (41 41.5 42.5 43) (42 42.5 43.5 44) and there Fuzzy demand of the product and destination are (69 69.5 70.5 71) (29 29.5 30.5 31) (49 49.5 50.5 51).

Solution:

The fuzzy Transportation problems are given in Table-1

Table-1

Source	D1	D2	D3	Supply
S1	(4, 4.5, 5.5, 6)	(6, 6.5, 7.5, 8)	(7, 7.5, 8.5, 9)	(69,69.5, 70.5, 71)
S2	(3,3.5, 4.5, 5)	(3,3.5, 4.5, 5)	(5, 5.5, 6.5, 7)	(29,29.5, 30.5, 31)
S3	(5, 5.5, 6.5, 7)	(6, 6.5, 7.5, 8)	(6, 6.5, 7.5, 8)	(49,49.5, 50.5, 51)
De-mand	(64, 64.5, 65.5, 66)	(41, 41.5, 42.5, 43)	(42, 42.5, 43.5, 44)	

The fuzzy transportation problem can be formulated in the following mathematical form

Min Z =R(4,4.5,5.5,6)X11+R (6,6.5,7.5,8)X12 +R (7,7.5,8.5,9)X13 + R (3,3.5,4.5,5) X 21 +R (3,3.5, 4.5, 5)X22 +R(5, 5.5, 6.5, 7)X23+ R (5, 5.5, 6.5, 7)X31+ R (6, 6.5, 7.5, 8) X32 + R(6, 6.5, 7.5, 8) X33

$$R(\overline{a})\int_0^1 (0.5)(a_\alpha^L\ ,a_\alpha^U)\,da$$

where

(a_α^L, a_α^U) { (b-a) α + a, d-(d-a)α}

(a_α^L, a_α^U) =

R(4,4.5,5.5.,6)=

$\int_0^1 (0.5)\ (4.5-4)\ \alpha + 4, 6-(6-5.5)\ \alpha$

= 5

R(6,6.5,7.5,8) =

$\int_0^1 (0.5)(6.5-6)\ \alpha\ + 6, 8-(8-7.5)\ \alpha$

= 7

Similarly,

R(7, 7.5, 8.5, 9) = 8 R(3, 3.5, 4.5, 5) = 4
R(3, 3.5, 4.5, 5) = 4 R(5, 5.5, 6.5, 7) = 6
R(5, 5.5, 6.5, 7) = 6 R(6, 6.5, 7.5, 8)= 7
R(6, 6.5, 7.5, 8) = 7

Rank of all Supply : R(69,69.5, 70.5, 71) = 70, R(29,29.5, 30.5, 31)=30, R(49,49.5, 50.5, 51) = 50

Rank of Demand :R(64, 64.5, 65.5, 66)= 65,
R(41, 41.5, 42.5, 43)= 42,
R(42, 42.5, 43.5, 44) = 43

Table – 2 (After Ranking)

Sources	D1	D2	D3	Supply
S1	5	7	8	70
S2	4	4	6	30
S3	6	7	7	50
Demand	65	42	43	

Table – 3

Sources	D1	D2	D3	Supply
S1	5 (65)	7 (5)	8	70
S2	4	4 (30)	6	30
S3	6	7 (7)	7 (43)	50
De-mand	65	42	43	

Table - 4

Sources	D1	D2	D3	Supply
S1	(4, 4.5, 5.5, 6) **(64,64.5, 65.5,66)**	(6, 6.5, 7.5, 8) **(4,4.5,5.5 ,6)**	(7, 7.5, 8.5, 9)	(69,69.5 , 70.5, 71)
S2	(3,3.5, 4.5, 5)	(3,3.5, 4.5, 5) **(29,29.5, 30.5,31)**	(5, 5.5, 6.5, 7)	(29,29.5 , 30.5, 31)
S3	(5, 5.5, 6.5, 7)	(6, 6.5, 7.5, 8) **(6,6.5,7.5 ,8)**	(6, 6.5, 7.5, 8) **(42,42.5, 43.5,44)**	(49,49.5 , 50.5, 51)
Demand	(64, 64.5, 65.5, 66)	(41,41.5, 42.5,43)	(42, 42.5, 43.5, 44)	

In this example it has show that the total optimal cost obtains by hour's method demands so that obtained by defuzzying the total four optimal solutions by applying Robust Ranking Method for the fuzzy transportation problem with fuzzy optimal solution

Min Z = 65 x 5 + 5 x7 + 30 x 4 + 7 x 7 + 43 x 7
= Rs. 830

6. Conclusion

The transportation cost is considered as imprecise fuzzy numbers in this paper. Here, the fuzzy transportation has been changed into crisp transportation using Robust ranking Method. Later, an optimal solution has been originated from crisp and fuzzy optimal total cost of given example. Moreover, using Robust ranking Method and Zero suffix method can conclude that the solution of fuzzy problem obtained more accurately and effectively.

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