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Minimization of fuzzy assignment model using Robust Ranking technique with trapezoidal fuzzy numbers

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Abstract

Assignment problem is the special case of linear programming allocation problem. Consider the situation of assigning m jobs to n machines. The cost of assigning the jobs i = 1,2,…., m to a machine j = 1,2,…, n is $C_{ij.}$ In this paper C_{ij} has been consider to be trapezoidal number denoted by C_{ij} which are more precise in nature. Robust ranking method has been used for ranking the fuzzy numbers. The Hungarian (One's assignment) method has been applied to solve the fuzzy assignment problem. An illustrative numerical example is provided to demonstrate the effectiveness of the fuzzy assignment problems.

Keywords: Hungarian method, Robust ranking method, Trapezoidal numbers.

1. Introduction

The assignment model is the special case of transportation where the number of sources equals the number of destinations and each capacity and requirement value is exactly one unit. Here the job represents sources and machine represent destination. The cost of assigning job i to machine j is C_{ij}. The objective is to minimize the overall cost for the assignment schedule.

Let x_{ij} represents assignment of i^{th} job to j^{th} machine. Therefore,

$$x_{ij} = \begin{cases} 1, & \text{if the ith job is assigned to } j^{th} \text{ machine} \\ 0, & \text{otherwise} \end{cases}$$

c١١ x١١	c١٢ x١٢	---	c١n x١n
c٢١ x٢١	c٢٢ x٢٢	----	c٢n x٢n
----	----	----	----
cn١ xn١	cn٢ xn٢	----	cnn xnn

If x_{ij} value is missing in a particular cell, no assignment is made between the pair of job and machine in question.

If the assignment is made, then we have two types of cells: Assignment cells for which x_{ij}=1 and non-assignment cells for which x_{ij} =0

The objective function is to minimize

$$\text{Min z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints,

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2,.........n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1,2,.........n$$

and x_{ij} = 0 or 1 for all i,j.

1.1 Fuzzy assignment problem:

In 1965, Lotfi Zadeh has introduced fuzzy sets which provide as a new mathematical tool to deal with uncertainty of information [3]. In this paper we provided a Hungarian and Robust ranking method to solve assignment problem with fuzzy cost $\widetilde{c_{ij}}$.

The objective of this paper to minimize the total cost subject to some crisp constraints also the objective function considered as the fuzzy numbers. The robust ranking method has been applied to rank the objective values of objective functions in order to transform the fuzzy assignment problem to a crisp situation. The Hungarian one's assignment method has been applied to find the optimal solution with fuzzy parameter to crisp version.

Lin and Wen solved the assignment problem with fuzzy interval number cost by a labeling algorithm [2]. Chen proved some theorems are proposed a fuzzy assignment model that considers all individuals to have same skills [5]. Dominance of fuzzy numbers can be explained by many ranking methods. Robust ranking method satisfies the properties of compensation, linearity and additivity. Zimmermann showed that solutions are obtained by fuzzy linear programming are always efficient [4]. R.Nagarajan and A.Solairaju [8] presented an algorithm for solving fuzzy assignment problems using Robust ranking technique with fixed fuzzy numbers.

In 1955, Kuhn proposed an algorithm for linear assignment problem known as Hungarian method [1]. Geetha et.al first formulated cost – time assignment problem has the multi criterion problem [9]. In existing literature, several researchers developed different metrologies for solving generalized assignment problem. Among this, one may refer to the works of Ross et.al [6]. Bai et.al proposed a method for solving fuzzy generalized assignment problem [7].

In this paper we applied a Robust Ranking techniques and Hungarian ones assignment method for solving fuzzy assignment problem with fuzzy jobs and fuzzy machines when cost edges are representing trapezoidal fuzzy numbers. Finally, feasibility of the proposed study is checked with a numerical example.

2. Preliminaries

Zadeh, first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life in 1965 [3].

2.1 Definition: A **fuzzy set** is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (i,e) $A = \{(x, \mu_A(x) ; x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \varepsilon X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition: (Trapezoidal fuzzy number): For a trapezoidal number A(x), it can be represented by A (a, b, c, d; 1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a) / (b-a), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ (d-x)/ (d-c), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

2.3 Definition: (α-cut of trapezoidal fuzzy number):

The α-cut of a fuzzy number A(x) is defined as

$$A(\alpha) = \{ x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$$

Addition of two fuzzy numbers can be performed as

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1+a_2, b_1+b_2, c_1+c_2)$$

Addition of two trapezoidal fuzzy numbers can be performed as

$(a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = (a_1+a_2,b_1+b_2, c_1+c_2,d_1+d_2)$.

3. Robust's Ranking Techniques

Robust's ranking technique [8] which satisfies costs, linearity, and additives properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust's Ranking Index is defined by

R (ã) = $\int_0^1 0.5\,(a^L_\alpha, a^U_\alpha)\,d\alpha$, where $(a^L_\alpha,\ a^U_\alpha)$ is the α – level cut of the fuzzy number ã.

In this paper we use this method for ranking the objective values. The Robust's ranking index R (ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property:

4. Hungarian Ones Assignment Method

For obtaining an optimal assignment schedule, the following steps may be suggested.

Step 1 : Prepare a cost matrix. If the cost matrix is not a square, add a dummy row (or dummy column) with zero cost elements. In a Minimization or Maximization case, find the minimum or maximum element of each row in the assignment matrix say a_i and write it on the right hand side of the

$$\begin{pmatrix} 1 & 2 & 3 & . & . & n \\ a_{11} & a_{12} & a_{13} & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2n} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ -- & - & - & - & - & - \\ a_{n1} & a_{n2} & a_{n3} & -- & - & a_{nn} \end{pmatrix} \begin{matrix} \\ a_1 \\ a_2 \\ - \\ - \\ - \\ a_n \end{matrix}$$

Step 2 : Divide a minimum element in each row from all the elements in its row. If there is atleast one 1's in each row and in column, stop here. Otherwise go to the next step.

Step 3 : Divide a minimum element in each column from all the elements its column in the above reduced matrix.

$$\begin{pmatrix} 1 & 2 & 3 & . & . & n \\ a_{11}/a_1 & a_{12}/a_1 & a_{13}/a_1 & . & . & a_{1n}/a_1 \\ a_{21}/a_2 & a_{22}/a_2 & a_{23}/a_2 & . & . & a_{2n}/a_2 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ -- & - & - & - & - & - \\ a_{n1}/a_n & a_{n2}/a_n & a_{n3}/a_n & -- & - & a_{nn}/a_n \end{pmatrix} \begin{matrix} \\ a_1 \\ a_2 \\ - \\ - \\ - \\ a_n \end{matrix}$$

Step 4 : In the above reduced cost matrix for an optimum assignment as follows,

(i) Examine the rows successively until a row with exactly one 1 is found. Matrix their 1 as□, indicating that an assignment will be made there. Mark all other 1s in its column 'X', indicates that they can be used until all rows have been examined. If there are more than one 1s, defer the decision.

(ii) Repeat the procedure given in (i) for the column.

(iii) Repeat (i) and (ii) successively until one of the following two occurs.

a. Each row and each column of a reduced cost matrix has one and only one assigned one. There may be no row and no column without assignment. In such a case, we have an optimal assignment.

b. Otherwise we go to the following steps.

Step 5 : Draw the minimum number of horizontal and vertical lines to cover all the zeros as explained below.

a. Mark the rows for which the assignment has not been made.

b. Mark the columns which have 1s in the marked rows.

c. Against the marked column, look for any assignment and mark that row.

d. Repeat steps (i) to (iii) until no more marking is possible.

e. Draw lines through all unmarked rows and marked columns.

Step 6 : Do he following steps.

a. Examine the element that do not have a line through them and select the smallest one.

b. Divide it from all the uncrossed elements (i.e elements that do not have a line through them) and multiply the same at the intersection of two lines.

c. Other elements remain unchanged.

Step 7 : Go to step 4 and repeat the procedure till an optimum assignment is achieved.

5. Numerical Example: (Trapezoidal Fuzzy Number)

Four different jobs can be done on four different machines and take down time costs are prohibitively high for change over. The matrix below gives the cost in rupees of producing job i on machine j.

A company has four sources S_1, S_2, S_3, S_4 and destinations D_1, D_2, D_3, D_4. The fuzzy transportation cost for unit quantity of product from i^{th} sources j^{th} destinations is C_{ij}.

Where Cij,

(15,15.5,16.5,17)	(9,9.5,10.5,11)	13,13.5,14.5,15)	(10,10.5,11.5,12)
(13,13.5,14.5,15)	(10,10.5,11.5,12)	(14,14.5,15.5,16)	(14,14.5,15.5,16)
(14,14.5,15.5,16)	(14,14.5,15.5,16)	(12,12.5,13.5,14)	(12,12.5,13.5,14)
(12,12.5,13.5,14)	(11,11.5,12.5,13)	(13,13.5,14.5,15)	(14,14.5,15.5,16)

Find the assignment of persons to jobs that will minimize the total fuzzy cost.

Solution: In conformation to model the fuzzy assignment problem can be formulated in the following

Min R(15,15.5,16.5,17) x_{11} + R(9,9.5,10.5,11)x_{12} + R(13,13.5,14.5,15)x_{13} + R(10,10.5,11.5,12)x_{14} + R(13,13.5,14.5,15)x_{21}+ R(10,10.5,11.5,12)x_{22} +R(14,14.5,15.5,16)x_{23} + R(14,14.5,15.5,16)x_{24} + R(14,14.5,15.5,16)x_{31}+ R(14,14.5,15.5,16)x_{32} + R(12,12.5,13.5,14)x_{33} + R(12,12.5,13.5,14)x_{34} +R(12,12.5,13.5,14)x_{41}+ R(11,11.5,12.5,13)x_{42}+ R(13,13.5,14.5,15)x_{43}+ R(14,14.5,15.5,16)x_{44}

Subject to

$x_{11}+x_{12}+x_{13}+x_{14} =1 \quad x_{11}+x_{21}+x_{31}+x_{41}=1$
$x_{21}+x_{22}+x_{23}+x_{24} =1 \quad x_{12}+x_{22}+x_{32}+x_{42}=1$
$x_{31}+x_{32}+x_{33}+x_{34} =1 \quad x_{13}+x_{23}+x_{33}+x_{43}=1$
$x_{41}+x_{42}+x_{43}+x_{44}=1 \quad x_{14}+x_{24}+x_{34}+x_{44}=1$
where $x_{ij} \in [0,1]$

Now we calculate R (15, 15.5, 16.5, 17) by applying Robust ranking method. The membership function of the trapezoidal fuzzy number (15, 15.5, 16.5, 17) is

$$\mu(x) = \begin{cases} \dfrac{x-15}{15.5-15}, & 15 \le x \le 15.5 \\ 1, & 15.5 \le x \le 16.5 \\ \dfrac{17-x}{17-16.5}, & 16.5 \le x \le 17 \end{cases}$$

0 , otherwise

The α – cut of the fuzzy number (15, 15.5, 16.5,17) is $(a_{\alpha}^{L}, a_{\alpha}^{U})$ = (0.5α +15, 17- 0.5α) for which

$$R(\overline{a}) \int_0^1 (0.5)(a_{\alpha}^{L}, a_{\alpha}^{U})\, da$$

where

$(a_{\alpha}^{L}, a_{\alpha}^{U})$ { (b-a) α + a,d-(d-c)α}

$(a_{\alpha}^{L}, a_{\alpha}^{U})$ =

$$\int_0^1 (((b-a) \propto +a, (d-(d-c) \propto))d \propto$$

R(15, 15.5, 16.5,17)=

$$\int_0^1 (((15.5-15) \propto +15, (17-(17-16.5) \propto))d \propto$$

= 16

R(9,9.5,10.5,11) = =

$$\int_0^1 (((9.5-9) \propto +9, (11-(11-10.5) \propto))d \propto$$

= 10

Similarly,

R(13,13.5,14.5,15) = 14 R(10,10.5,11.5,12)= 11
R(13,13.5,14.5,15)= 14 R(10,10.5,11.5,12) = 11 R(14,14.5,15.5,16) = 15 R(14,14.5,15.5,16)= 15
R(14,14.5,15.5,16) = 15 R(14,14.5,15.5,16)= 15
R(12,12.5,13.5,14)=13 R(12,12.5,13.5,14) =12
R(12,12.5,13.5,14)=13 R(11,11.5,12.5,13) = 12
R(13,13.5,14.5,15)=14 R(14,14.5,15.5,16) = 15

We replace these values for their corresponding a_{ij}, In which result in a convenient assignment problem in the linear programming problem.

$$\begin{pmatrix} 16 & 10 & 14 & 11 \\ 14 & 11 & 15 & 15 \\ 15 & 15 & 13 & 12 \\ 13 & 12 & 14 & 15 \end{pmatrix}$$

We solve it by one assignment methods to get the following optimal solution

Step:1

In a minimization case find the minimum element of the each row in the assignment matrix say (a_i) write it on the right hand side of the matrix, then

				Row Min
16	10	14	11	10
14	11	15	15	11
15	15	13	12	12
13	12	14	15	12

Then divide each element of i^{th} row of the matrix by a_i, these operations create at least one 1's in each rows, in tern of 1's for each row and column do assignment, otherwise go to step 2.

				Row Min
16/10	10/10	14/10	11/10	10
14/11	11/11	15/11	15/11	11
15/12	15/12	13/12	12/12	12
13/12	12/12	14/12	15/12	12

				Row Min
1.6	1	1.4	1.1	10
1.27	1	1.38	1.36	11
1.25	1.25	1.08	1	12
1.08	1	1.16	1.25	12

Step 2 : Find the minimum element of each column in assignment matrix (b_j), write it below j^{th} column then divide the each element of the j^{th} column of the matrix by b_j. These operations create al least one 1s in each column.

	1.48	1	1.34	1.1
	1.17	1	1.25	1.36
	1.15	1.25	1	1
	1	1	1.07	1.25
min	1	1	1	1

Step 3: Make assignment in terms of 1s.

1.48	1	1.34	1.1
1.17	[1]	1.25	1.36
1.15	1.25	[1]	1
[1]	1	1.07	1.25

Step 4; Divide it from all the uncrossed elements (i.e elements that do not have a line through them) and multiply the same at the intersection of two lines. Other elements remain unchanged.

1.34	1	1.21	[1]
1.06	[1]	1.13	1.23
1.15	1.37	[1]	1
[1]	1.10	1.07	1.25

We can assign the 1s and the solution is (1, 3), (2, 2), (3,1) and (4, 4).

The fuzzy optimal total cost $\widetilde{a_{14}}$ + $\widetilde{a_{22}}$ + $\widetilde{a_{33}}$ + $\widetilde{a_{41}}$ = R(10,10.5,11.5,12) + R(10,10.5,11.5,12) + R(12,12.5,13.5,14)+R(12,12.5,13.5,14) = R(44,46,50, 52).

6. Conclusion

In this paper, the assignment cost has been considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Here, the fuzzy assignment problem has been converted into crisp assignment problem using Robust ranking indices [10] and Hungarian ones assignment has been applied to find an optimal solution. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

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