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SOLUTION OF STOCHASTIC COOPERATIVE INVENTORY MODELS BY INTUTIONISTIC FUZZY OPTIMIZATION TECHNIQUE

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Abstract

Inventory management studies to minimize the average total cost per unit time and to determine the quantity of the stocked material to be ordered. In this paper the demand follows normal distributive under deterministic, stochastic and mixed environment, respectively and thus the model becomes more realistic. Expected annual cost is measured, with varying defective rate. After that the item wise multi objective models with chance-constraints for both exponential and uniform demand are taken and the results are compared numerically both in fuzzy optimization and intuitionistic fuzzy optimization techniques. Objective of this paper is to establish that intuitionistic fuzzy optimization method is better than usual fuzzy optimization technique as expected annual cost of this inventory model is more minimized in case of intuitionistic fuzzy optimization method.

Keywords: cooperative game theory, inventory management, intuitionistic fuzzy optimization method , demand, chance-constraint, multi objective stochastic model, Economic order Quantity model.

1. Introduction

In conventional inventory models, uncertainties are treated as randomness and are handled by appealing to probability theory .However ,in certain situations uncertainties are due to fuzziness and these cases the fuzzy set theory, originally introduced by Zadeh is applicable.I n decision making process ,first, Bellman and Zadeh introduced fuzzy set theory Thaka et al.a pplied concept of fuzzy sets to decision making problems to consider the objectives as fuzzy goals over the α-cuts of a fuzzy constraints. Zimmerman showed the classical algorithms can be used in few inventory models.

Intuitionistic fuzzy sets (IFS) was introduced by K.Atanssovn seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that IFS can be used to simulate human decision making process and any activities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity. Atanossov also analysed intuitionistic fuzzy sets in a more explicit way. Atanassov discussed as open problems in Intuitionistic fuzzy sets theory. An interval valued Intuitionistic fuzzy sets as analyzed by Atanassov and Gargov. Angelvov implemented the optimization in an Intuitionistic fuzzy environment. Pramanik and Roy solved a vector optimization problem using an Intuitionistic fuzzy goal programming. Banerjee and Roy analyzed a probabilistic fixed order interval system by general fuzzy progrmming technique and Intuitionistic fuzzy optimization technique . Banerjee and roy also solved a stochastic inventory model with fuzzy cost components by fuzzy geometric and Intuitionistic fuzzy geometric programming technique. Mahapatra presented the redundancy optimization by Intuitionistic fuzzy multi-objective programming. Banerjee and Roy discussed the solution of a constrained stochastic model by fuzzy geometric and Intuitionistic fuzzy geometric programming.

As a single objective stochastic inventory problem, Paknejad et al's model is analyzed in this paper under deterministic,stochastic and mixed environment, respectively, where the lead time demand follows normal distribution. Expected annual cost is measured, with varying defective rate ,in deterministic environment. After that an item wise multi objective models with chance-constraints for both exponential and uniform lead time demand are taken and the results are compared numerically both in fuzzy optimization and Intuitionistic fuzzy techniques. Objective of this paper is to estabilish that Intuitionistic fuzzy optimization technique as expected annual cost of this inventory model is more minimized in case of model is more minimized Intuitionistic fuzzy optimization method. Necessary numerical illustrations are also given below.

2 Classical Cooperative Inventroy Games

An n-person economic order quantity (EOQ) situation is a well-known and simple operational research model constructed by an inventory situation.I t considers an agent who makes orders of a certain good that he sells. The demand of agent i that must be fulfilled is deterministic and equals to di units per time (di ≥0).The cost of keepingone unit of this good per unit in stock is hi(hi ≥ 0). If the fixed ordering cost and the leading time, the time between the placement of an order and the delivery of that order, are deterministic. Recall that the average inventory cost per unit time is a function of the order size Qi , which is given by

$$C(Q_i) = a\frac{d_i}{Q_i} + h_i\frac{Q_i}{2}$$

And the optimal order size

$$\hat{Q}_i = \sqrt{\frac{2ad}{h_i}}$$

2.1 Cooperative fuzzy inventory games

The most important critic trade for deterministic EOQ model is the notion that parameter values are accurately known. However ,in real life situations neither the cost parameters nor demand rate are known exactly.

In this study if the parameters of an EOQ model not known accurately expect the demand rate, then we use the fuzzy numbers. Parameters used in a fuzzy EOQ model are defined as follows.

3 Notations:

[a1,a2] :fuzzy number for ordering cost with lower bound a1 and upper bound a2

[h1,h2]: fuzzy number for holding cost with lower bound h1 and upper bound h2

D : Deterministic demand rate

$\overline{AC}$: Average fuzzy total inventory cost

Q: order quantity for each period

The total cost function $\overline{AC}$ (Q) when ordering the quantity Q per order is

$$\overline{AC}(Q) = (a_1 + a_2)\frac{D}{2Q} + (h_1 + h_2)\frac{Q}{4}$$

When minimizing the average costs over all Q ≥ 0 , The optimum order size Q is

$$Q = \sqrt{\frac{2(a_1 + a_2)D}{h_1 + h_2}}$$

3.1 Single objective stochastic inventory model

$$\overline{AC}(Q) = (a_1 + a_2)\frac{D}{2Q} + (h_1 + h_2)\frac{Q}{4} \quad ,Q>0$$

It is the stochastic model, which minimizes the expected annual cost.

3.2 Multi item stochastic inventory model

Multi item is analyzed here firstly in deterministic environment and after that it is discussed in stochastic and mixed environment also.

4 Model with deterministic budget and storage

To solve the problem in equation as a MISIM it can be reformulated as

$$\min \overline{AC}(Q_1, Q_2, \ldots\ldots\ldots Q_i) = (a_{1i} + a_{2i})\frac{D_i}{2Q_i} + (h_{1i} + h_{2i})\frac{Q_i}{4}$$

$$\textit{subject to} \quad \sum_{i=1}^{n} f_i\, Q_i \le F$$

$$\sum_{i=1}^{n} pi\,Qi \le B \qquad , Q_i > 0 \ \forall i = 1,2,3,\ldots\ldots n$$

Here for the ith item (i=1,2,.....n)

Pi = price per unit item

fi= floor space available per unit

N= number of item

F=available floor space

B=total budget

2. Model with stochastic budget and fuzzstorage:

$$\min \overline{Ac}(Q_1, Q_2, Q_3, ...Q_n) = (a_{1i} + a_{2i})\frac{D_i}{2Q_i} + (h_{1i} + h_{2i})\frac{Q_i}{4}$$

$$sub\ \ to\ \ \ poss\ \ [\sum_{i=1}^{n} f_i\, Q_i = \overline{F}\,] \geq \sim$$

$$\sum_{i=1}^{n} P_i\, Q_i \leq \overline{\mathbf{B}} \qquad Q_i > 0\ \forall i = 1,2,.....n \qquad 0 < \sim < 1$$

3.Multi-objective stochastic inventory:

$$\min \overline{Ac}_i\,(Q_i) = (a_{1i} + a_{2i})\frac{D_i}{2Q_i} + (h_{1i} + h_{2i})\frac{Q_i}{4} \qquad , Q_i \geq 0\ \forall\, i = 1,2,.....n$$

4.Model with deterministic budget and storage:

$$\min \overline{Ac}_i\,(Q_i) = (a_{1i} + a_{2i})\frac{D_i}{2Q_i} + (h_{1i} + h_{2i})\frac{Q_i}{4} \qquad , Q_i \geq 0\ \forall\, i = 1,2,.....n$$

$$sub\ \ to\ \ \ [\sum_{i=1}^{n} f_i\, Q_i = F\,] \geq \sim$$

$$\sum_{i=1}^{n} P_i\, Q_i \leq B \qquad Q_i > 0\ \forall i = 1,2,.....n \qquad 0 < \sim < 1$$

.

5.Model with stochastic budget and storage:

$$\min AC_i\,(Q_i) = (a_{1i} + a_{2i})\frac{D_i}{2Q_i} + (h_{1i} + h_{2i})\frac{Q_i}{4} \qquad ,$$

$$sub\ \ to\ \ poss\ \ [\sum_{i=1}^{n} \overleftrightarrow{f}_i\, Q_i = \overleftrightarrow{F}\,] \geq \sim$$

$$\sum_{i=1}^{n} \overleftrightarrow{P}_i\, Q_i \leq \overleftrightarrow{B} \qquad Q_i > 0\ \forall i = 1,2,.....n$$

Fuzzy non-linear programming (FNLP) technique to solve multi-objective non-linear progrmming problem

A multi-objective non-linear programming (MONLP) or vector minimization problem(VMP) may be taken in the

$$\min f(x) = (f_1(x), f_2(x), ...f_k(x))^T$$

$$subject to\ \ x \in X = \{x \in R^n : g\ j(x) \leq or = \geq b_j\ \ for\ \ j = 1,2,....m\}$$

$$and\, l_i \leq x \leq u_i\ \ (i = 1,2,.......n)$$

following form:

zimmermann showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

6.To solve MONLP problem, following steps are used:

Step 1:

Solve the MONLP of equation as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2:

From the result of step 1, determine the corresponding values for every objective at each solution derived. ith the values of all objectives at each idle solution, pay-off matrix can be formulated as follows:

$$\begin{bmatrix} f_1(x^1) & f_2(x^1) & f_3(x^1) & ...... & f_k(x^1) \\ f_1(x^2) & f_2(x^2) & f_3(x^2) & .... & f_{k(}x^2) \\ .... & .... & .... & .... & .... \\ f_1(x^k) & f_2(x^k) & f_3(x^k) & .... & f_k(x^k) \end{bmatrix}$$

Here x1,x2,.......xk are the ideal solutions of the objective functions f1(x),f2(x),.......fk(x) respectively.

So Ur = max{ fr(x1),fr(x2),.........fr(xk)} and Lr = min{fr(x1),fr(x2),.......fr(xk)}

Lr, Ur be lower and upper of the rth objective functions fr(x) r =1,2,......k

Step3:

Using aspirtion level of each objective of the MONLP of equation may be written as follows:

Find x so as to satisfy

Fr(x) ≤ Lr ,x εX ,r =1,2,.....k

Here objective function of equation are considered as fuzzy constraints. These type of fuzzy constraints can be quantified by eliciting a corresponding membership function:

μr(fr(x)) = 0 if fr(x) ≥Ur

= μr(fr(x)) if Lr ≤ fr(x)≤ U

r =1,2,........k

= 1 if fr(x) ≤ Lr

Having elicited the membership functions μr(fr(x)) for r=1,2,.....k introduce a general aggregation function

μD(x) = G(μ1(f1(x),μ2(f2(x),.......,μk(fk(x)))

so a fuzzy multi-objective decision making problem can be defined as

Max μD(x)

Subject to xε X

Here we adopt the fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach) .

In this case equation is known as FNLP..

Then the problem of equation , using the membership functions as in equation according to min-operator is reduced to:

Max α

Subject to μi(fi(x)) ≥ α for i=1,2,.....k xε X , α ε[0,1]

Step 4:

Solve the equation to get optimal solution.

7.Formultion of Intuitionistic fuzzy optimization:

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other ,then IF sets can be used as a more general tool for describing uncertainty

To minimize the degree of acceptance the degree of rejection of IF objectives and constraints, we can write:

Max μi(X) , Xε R ,i=1,2,.....K+n

Min υi(X) , Xε R ,i=1,2,.....K+n

Subject to υi(X) ≥ 0

μi(X) ≥ υ i(X)

μi(X) + υi(X)<1

X ≥ 0

Where μi(X) denotes the degree of membership function of X to the ith IF sets υi(X) denotes the degree of non-membership (rejection) of X from the ith IF sets.

8.An intuitionistic fuzzy approach for solving MOIP with linear membership and non-membership functions:

To define the membership function of MOIM problem, let Lk acc and Uk acc be the lower and upper bounds of the kth objective function. These values re determined as follows: Calculate the individual minimum value of each objective function as a single objective IP subject to the given set of constraints. Let X1,X2,.......Xk be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed here that t least to of these solutions are different for which the kth objective function has different bounded values. For each objective, find lower bound (minimum value) Lk acc and the upper bound (maximum value)

Uk acc .But in intuitionistic fuzzy optimization (IFO),the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of MOIM problem, let Lk rej and Uk rej be the lower and upper bound of the objective function Zk(X) where Lk acc≤ Lk rej ≤Ukrej≤Ukacc .These values are defined as follows:

The linear membership function for the objective Zk(X) is defined as:

$$«_k(Z_k(X)) = \begin{cases} 1 & if\ \ Z_k(X) \le L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(X)}{U_k^{acc} - L_k^{acc}} & if\ L_k^{acc} \le Z_k(X) \le U_k^{acc} \\ 0 & if\ Z_k(X) \ge U_k^{acc} \end{cases}$$

$$«_k(Z_k(X)) = \begin{cases} 1 & if\ \ Z_k(X) \le U_k^{rej} \\ \dfrac{Z_k(X) - L_k^{rej}}{U_k^{acc} - L_k^{acc}} & if\ L_k^{rej} \le Z_k(X) \le U_k^{rej} \\ 0 & if\ Z_k(X) \ge L_k^{acc} \end{cases}$$

Then the solution of the MIOM problem is summari

. Step 1:

Pick the first objective function and solve it as a single objective IP subject to the constraint,continue the process K-times for K different objective functions. If all the solutions i.e. X1=X=......Xk i=1,2,.....m; j=1,2,......n same ,then one of them is the optimal compromise solution and go to step 6.Otherwise go to step 6.Otherwise go to step2.However, this rarely happens due to the conflicting objective functions.

Then the intuitionistic fuzzy goals take the form Zk(X)≤ Lk(X), k=1,2,.....K

Step 2

To build membership function, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step 1,e find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

$$\begin{bmatrix} z_1(x^1) & z_2(x^1) & z_3(x^1) & ...... & z_k(x^1) \\ z_1(x^2) & z_2(x^2) & z_3(x^2) & .... & z_{k(}x^2) \\ .... & .... & .... & .... & .... \\ z_1(x^k) & z_2(x^k) & z_3(x^k) & .... & z_k(x^k) \end{bmatrix}$$

Step 3:

From step 2,we find the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

Uk acc= max(Zk(Xr)) and Lk acc= min(Zk(Xr)) ,1≤ r≤ k

For linear membership functions,

Lk rej = Lk acc + t(Ukacc –Lkacc) here 0 < t <1

Uk rej = Uk acc + t(Ukacc –Lkacc) for t=0

Step 4:

Construct the fuzzy programming problem of equation and find its equivalent LP problem of equation

Step 5:

Solve equation by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the k objective functions at these optimal compromise solutions.

9.Numerical example:

Demand rates(items/per year) =4128

Ordering costs/year =[380,400]

Holding costs/year = [0.4,0.5]

Q = 2674.92

AC(Q) = 1203.71

10. Conclusion:

We assume that lead-time demand for the period for the ith item is a random variable that follows thus this model becomes more practical and realistic. In case of intuitionistic fuzzy optimization not only the membership function is maximized but also, the non-membership function is minimized. That is why intuitionistic fuzzy optimization improves the solution than fuzzy optimization. Thus expected annual cost is more minimized in case of intuitionistic fuzzy optimization than the usual fuzzy optimization technique. This model can also be extended taking lead-time demand as fuzzy random variables.

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